Basic solutions of liquid metal mixed convection in a horizontal duct under strong magnetic field and horizontal temperature gradient

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Abstract

Steady-state solutions of liquid metal mixed convection in a horizontal duct with electrically insulated walls under strong magnetic field and horizontal temperature gradient are computed by Newton method with finite element discretization through the high level integrated software FreeFem++. When a zero streamwise pressure gradient is applied, the longitudinal flow structure with concentrated jets near the corners of the cross-section for vertical magnetic field is verified for large Hartmann number and large Grashof number. Under both vertical and horizontal magnetic field, applied streamwise pressure gradient would break up-down symmetry of flow structure (e.g. the longitudinal velocity is no longer odd with respect to the vertical coordinate) . With the increase of the streamwise pressure gradient, the flow struture is changed greatly. The influence of the bouyancy force becomes so weak that Hartmann boundary layer appears for enough large streamwise pressure gradient.

Introduction

The interaction of the moving liquid-metal fluid with a magnetic field gives rise to a rich variety of phenomena. On the one hand, under a strong magnetic field the mean flow profile may create inflexion points (Kakutani 1964), shear layers (Lehnert 1952) and jets (Hunt 1965), producing instability of free shear flow type. On the other hand, strong magnetic field tends to damp three-dimensional perturbations by suppressing the variation in the magnetic field direction; the action of strong magnetic field would suppress the production of turbulence and make the transition from laminar flow to turbulence occur at much higher Reynolds number (Shatrov & Gerbeth 2010).

In currently designed liquid metal (Li or PbLi) blankets for future nuclear fusion reactors, mixed (combined natural and forced) thermal convections in long duct-shaped conduits under strong magnetic field are the main flow scheme (Molokov et al. 2007, Smolentsev et al. 2008). For such complex mixed convections, conventional turbulence is most likely fully suppressed by such a strong magnetic field, while great temperature gradient along the blanket ducts makes the flow more unstable. Some attempts of computational analysis of the thermal convection phenomena in configurations corresponding to specific designs of fusion reactor blankets have been performed by direct numerical simulations (Mas de les Valls et al. 2011, Mistrangelo & Bühler 2013, Vetcha et al. 2013). These time-dependent DNS typically face great obstacles of high requirements on numerical resolution, which should be satisfied in order to accurately obtain flow features, such as the MHD boundary layers.

In this paper, steady-state liquid metal mixed convection in a horizontal duct with electrically insulated walls under strong magnetic field and horizontal temperature gradient are investigated by Newton method along with finite element scheme. Through the numerical computation, all kinds of interesting flow structure with different streamwise pressure gradient for large Hartmann number and large Grashof number are presented and analyzed.

Physical Model

The liquid metal is suitably regarded as an incompressible, electrically conducting Newtonian viscous fluid with constant kinematic viscosity ν and thermal diffusivity κ. A liquid metal with a constant electric conductivity σ flows along a horizontal rectangular duct subject to a horizontal temperature gradient ∇T and an external constant magnetic field $\mathbf{B}_0$, as shown in Fig. 1. Boussinesq approximation is used for the representation of buoyancy force

$$\mathbf{f} = (\rho - \rho_0)\mathbf{g}, \quad \rho = \rho_0[1 - \beta(T - T_0)]$$

Here, β is the thermal expansion coefficient and the subscript 0 is denoted as a reference value. The magnetic field **B** is the sum of the applied magnetic field $\mathbf{B}_0$ and the induced field **b**. Since in most laboratory experiments the magnetic Reynolds number is very small, the magnetic field remains almost unperturbed and the induced magnetic field **b** is then negligible.

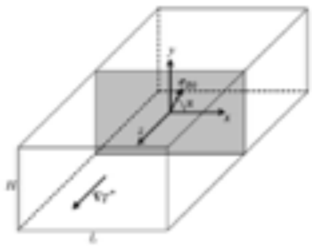


Figure 1. Flow chart of a liquid metal mixed convection in a horizontal duct under strong magnetic field and horizontal temperature gradient.

By assuming a low magnetic Reynolds number, the quasi-static approximation (Davidson 2001) is used and the governing equations can be reduced to the Navier-Stokes system

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right] = -\nabla p + \rho\nu\nabla^2\mathbf{v} + \mathbf{f} + \mathbf{j}\times\mathbf{B}_0,$$

$$\nabla\cdot\mathbf{v} = 0,$$

Here, $\mathbf{v}=(u, v, w)$ denotes the velocity vector, p the pressure, ρ the fluid mass density, ν its kinematic viscosity, and volumetric buoyancy force **f** and Lorentz force term $\mathbf{j}\times\mathbf{B}_0$. The induced electric current density **j** is given by Ohm's law:

$$\mathbf{j} = \sigma(-\nabla\phi + \mathbf{v}\times\mathbf{B}_0).$$

Here, φ is the the electric potential. By neglecting displacement currents and assuming the fluid to be electrically neutral, it follows that the currents are solenoidal, i.e.

$$\nabla\cdot\mathbf{j} = 0,$$

which is Combined with above Ohm's law, an possion equation for the electric potential is obtained:

$$\nabla^2\phi = \nabla\cdot(\mathbf{v}\times\mathbf{B}_0).$$

The balance of the total energy in a volume element leads to a convection diffusion equation for temperature T of the form

$$\rho c_p \left[\frac{\partial T}{\partial t} + (\boldsymbol{v}\cdot\nabla)T\right] = \nabla\cdot(\lambda\nabla T) + \frac{1}{\sigma}\boldsymbol{j}^2 + \Phi + Q,$$

Where c_p is the specific heat capacity, λ the thermal conductivity, $\mathbf{j}^2$ the loss of magnetic energy due to Joule dissipation, Φ the loss of kinetic energy due to viscous dissipation, and Q other sources of volumetric energy release like nuclear radiation or chemical reactions. By neglecting all energy dissipation except the Fourier diffusion term, the temperature equation is reduced into

$$\frac{\partial T}{\partial t} + (\mathbf{v}\cdot\nabla)T = \kappa\nabla^2 T.$$

Here, κ is the thermal diffusivity.

The boundary conditions at the duct walls are the no-slip conditions for velocity

$$u = v = w = 0$$

perfect electric insulation

$$\frac{\partial\phi}{\partial n} = 0,$$

and fixed linear temperature distribution

$$T = (\nabla T)x$$

length	H
time	H^2/ν
velocity	ν/H
pressure	$\rho_0\nu^2/H^2$
temperature	$(\nabla T)H$
electric potential	νB_0
electric current	$\sigma\nu B_0/H$

Table 1. The characteristic scales for different physical quantities.

The governing equations can be further dimensionalized by the characteristic scales for different physical quantities shown in Table 1. Then the dimensionless governing equations can be written as

$$\frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p + \nabla^2\mathbf{v} + Gr\ T\mathbf{e}_y + Ha^2\mathbf{j}\times\mathbf{e}_{B_0},$$

$$\frac{\partial T}{\partial t} + (\mathbf{v}\cdot\nabla)T = \frac{1}{Pr}\nabla^2 T.$$

$$\nabla\cdot\mathbf{v} = 0,$$

$$\mathbf{j} = -\nabla\phi + \mathbf{v}\times\mathbf{e}_{B_0}.$$

$$\nabla\cdot\mathbf{j} = 0.$$

Here, the non-dimensional parameters are the Hartmann number, the Grashof number, the Prandtl number and aspect ratio of width and height, as follows

$$Ha = B_0 H\sqrt{\sigma/\rho_0\nu}, \quad Gr = g\beta(\nabla T)H^4/\nu^2,$$

$$Pr = \nu/\kappa, \quad l = L/H.$$

We look for a basic solution whose velocity and potential are independent of streamwise coordinate, as follows

$$\mathbf{v} = \mathbf{v}(x,y) = (\vec{U}(x,y), W(x,y)),\ T = z + \theta(x,y),\ \phi = \phi(x,y).$$

Note here the temperature except linear part is also assumed to be independent of streamwise coordinate. The pressure gradient is assumed to be a constant, then it is a linear function of streamwise coordinate z from the streamwise momentum equation,

$$\partial^2 p/\partial z^2 = 0 \Rightarrow p = D(x,y)z + \Pi(x,y).$$

from non-streamwise momentum equations we have

$$\nabla_s(\partial p/\partial z) = Gr\ \mathbf{e}_y \Rightarrow D(x,y) = Gr\,y + C.$$

Thus the pressure distribution is given by

$$p = Gr\,yz + \Pi(x,y) + Cz.$$

Here, $\boldsymbol{C}$ is the dimensionless pressure gradient along the streamwise direction.

Substituting these representations for velocity, pressure, temperature and potential into the govering equations, we would make the basic solution satisfy

$$\frac{\partial W}{\partial t} + (\vec{U}\cdot\nabla_s)W = -Gr\ y - C + \nabla_s^2 W + Ha^2 F_z,$$

$$\frac{\partial\vec{U}}{\partial t} + (\vec{U}\cdot\nabla_s)\vec{U} = -\nabla_s\Pi + \nabla_s^2\vec{U} + Gr\ \theta\mathbf{e}_y + Ha^2\vec{f},$$

$$\frac{\partial\theta}{\partial t} + (\vec{U}\cdot\nabla_s)\theta + W = \frac{1}{Pr}\nabla_s^2\theta,$$

$$\nabla_s^2\cdot\vec{U} = 0,$$

$$\vec{F} = (-\nabla\phi + \mathbf{v}\times\mathbf{e}_{B_0})\times\mathbf{e}_{B_0},$$

$$\nabla^2\phi = \mathbf{e}_{B_0}\cdot(\nabla\times\mathbf{v}).$$

Here, $\vec{U} = (U, V)$ and $\vec{F} = (\vec{f}, F_z)$ with $\vec{f} = (F_x, F_y)$.

Numerical computations

When Prandtl number is zero, stream-vorticity equations (can be derived easily) are decoupled from the streamwise equation due to

$$\nabla_s^2\theta = 0 \Longrightarrow \theta = 0,$$

The problem admits a steady solution in which only the longitudinal component of the velocity is non-zero, then the governing equations and boundary conditions are thus reduced to a linear system as follows

$$-Gr\ y - C + \nabla_s^2 W + Ha^2 F_z = 0$$

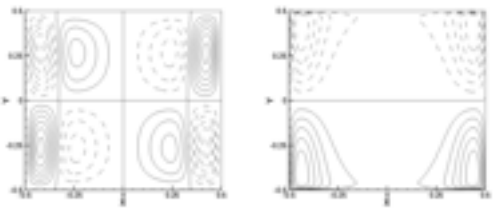
$$\nabla_s^2\phi = \left(\frac{\partial W}{\partial y}\cos\alpha - \frac{\partial W}{\partial x}\sin\alpha\right)$$

$$W|_\Gamma = \frac{\partial\phi}{\partial n}|_\Gamma = 0.$$

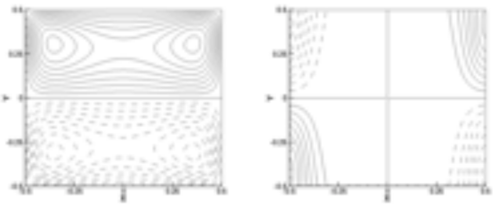
In the case of a non-zero Prandtl number, the mixed convection problem does not admit parallel basic solution and the full non-linear system of governing equations should be solved numerically. Two different methods can be used for the computation of the steady-state solutions: either the governing equations satisfying the basic solution independent of streamwise coordinate are integrated in time until the steady state is reached, or the system of stationary equations by assuming the time derivatives to be zero are solved by Newton method. Here the Taylor-Hood finite element method is used to discretization for the basic flow equations, and Newton method is adopted to obtain the steady-state solutions by using the basic solution for zero-Prandtl number as initial guess. A high level integrated software FreeFem++ to solve partial differential equations with finite element methods is used for quick realization of the finite element discretization and then the calculation of the steady-state solutions. Two cases for different directions of the magnetic field are considered for different pressure gradient $\boldsymbol{C}$ with the parameters Pr=0.1, Gr=10000, Ha=100, l=1 which represent enough large temperature gradient and magnetic intensity.

1. Vertical direction of the magnetic field

In case of the vertical magnetic field, the zero pressure gradient ($\boldsymbol{C}$=0) is first considered, while streamlines of the velocity and isolines of the longitudinal velocity, temperature and electric potential in the cross-section are plotted in Fig.2. For the zero pressure gradient, the average streamwise velocity is zero due to its up-down anti-symmetry which can also be deduced from the symmetries of steady-state governing equations. From Fig.2, four pars of vortical rolls appear along the horizontal direction and the longitudinal flow is concentrated in jets near the corners of the cross-section whereas the core of the duct is nearly zero flat. Upper jets are negative while downside jets are positive. The temperature field is also up-down anti-symmetry and the maximum value is located at the center of each quarter of the cross-section. The large gradient of electric potential is found near the corners of the cross-section. Our results for the streamlines of velocity and isolines of the temperature are exactly the same as those calculated by Lyubimova et al. (JFM 2009).



(a) (b)

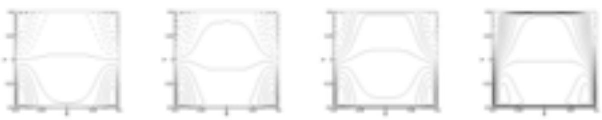


(c) (d)

Figure 2. Streamlines of the velocity (a) and isolines of the longitudinal velocity (b), temperature (c) and electric potential (d) in the cross-section for vertical magnetic field with ***C***=0, *Pr*=0.1, *Gr*=10000, *Ha*=100, *l*=1. (solid lines for positive values and dashed lines for negative values)

With the increase of the streamwise pressure gradient, the up-down symmetry of flow structure is broken obviously as shown in Fig.3. From the figures of streamlines of the velocity, it is easily found that two pairs of rolls are combined into two large rolls, while the other two pairs of rolls become smaller and finally disappear. From isolines of the longitudinal velocity, it can be seen that the lower two jet structures extend to the center region, and upper two jets become smaller and weaker. When the streamwise pressure gradient is large enough (not shown here), the flow will be changed into a flug-like structure, and the influence of bouyancy force will be so weak that the Hartmann boundary layers will appear near the wall vertical to the magnetic field. From isolines of the temperature, it can be seen that the lower negative temperature distribution becomes a large cell. With the increase of the streamwise pressure gradient, the large cell extends into upper region will the upper origin structrures become smaller and are limited to the region near the corners of the cross-section. Again when the streamwise pressure gradient is large enough, the negative cell will occupy the whole cross-section and the positive structures will diappear.

(a) Streamlines of the velocity (***C***=-10,-20,-40,-80 from left to right)



(b) Isolines of the longitudinal velocity (***C***=-10,-80,-320,-1000 from left to right)

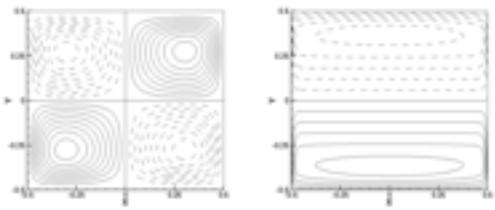
(c) Isolines of the temperature (***C***=-20,-40,-80,-160 from left to right)

Figure 3. Evolution of Streamlines of the velocity and isolines of the longitudinal velocity and temperature in the cross-section with the increase of the streamwise pressure gradient for vertical magnetic field. (*Pr*=0.1, *Gr*=10000, *Ha*=100, *l*=1.)

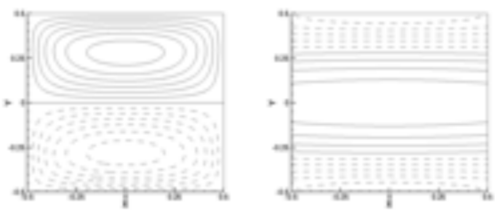
2. Horizontal direction of the magnetic field

In case of the horizontal magnetic field, the zero pressure gradient (***C***=0) is also first considered, and streamlines of the velocity and isolines of the longitudinal velocity, temperature and electric potential in the cross-section are plotted in Fig.4. It is easily seen from the streamlines of the velocity that there exist four rolls in the four quarter. The average streamwise velocity is zero due to its up-down anti-symmetry of the longitudinal velocity which has a counter flow structure. The velocity distribution is similar to that of the longitudinal velocity except that the negative temperature is located at the lower part. It is also found that the isolines of the electric potential are nearly parallel to the horizontal direction.

Under non-zero streamwise pressure gradient, the up-down symmetry is also broken for the horizontal magnetic field as shown in Fig.5. It is easily seen that with the increase of the streamwise pressure gradient, the lower structures becomes larger while the upper structure becomes smaller. Similar to the case of vertical magnetic field, when the streamwise pressure gradient is large enough, the lower structure will occupy the whole cross-section. However, this process needs much more streamwise pressure gradient compared to the situation of vertical magnetic field.

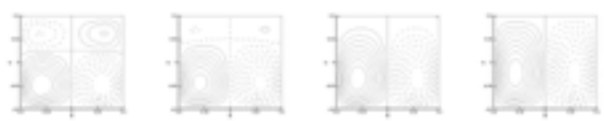


(a) (b)



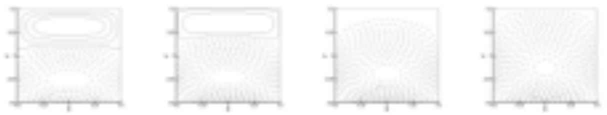
(c) (d)

Figure 4. Streamlines of the velocity (a) and isolines of the longitudinal velocity (b), temperature (c) and electric potential (d) in the cross-section for horizontal magnetic field with ***C***=0, *Pr*=0.1, *Gr*=10000, *Ha*=100, *l*=1. (solid lines for positive values and dashed lines for negative values)



(a) Streamlines of the velocity (***C***=-500,-1000,-2000,-4000 from left to right)

(b) Isolines of the longitudinal velocity (***C***=-500,-1000,-2000,-4000 from left to right)



(c) Isolines of the temperature (***C***=-500,-1000,-2000,-4000 from left to right)

Figure 5. Evolution of Streamlines of the velocity and isolines of the longitudinal velocity and temperature in the cross-section with the increase of the streamwise pressure gradient for horizontal magnetic field. (*Pr*=0.1, *Gr*=10000, *Ha*=100, *l*=1.)

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